Syllogistic Reasoning in Fuzzy Logic and its Application to Usuality and Reasoning with Dispositions

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Abstract—**A fuzzy syllogism in fuzzy logic is defined to be an inference schema in which the major premise, the minor premise and the conclusion are propositions containing fuzzy quantifiers. A basic fuzzy syllogism in fuzzy logic is the intersection/product syllogism. Several other basic syllogisms are developed that may be employed as rules of combination of evidence in expert systems. Among these is the consequent conjunction syllogism. Furthermore, we show that syllogistic reasoning in fuzzy logic provides a basis for reasoning with dispositions; that is, with propositions that are preponderantly but not necessarily always true. It is also shown that the concept of dispositionality is closely related to the notion of usuality and serves as a basis for what might be called a theory of usuality—a theory which may eventually provide a computational framework for commonsense reasoning.**

I. INTRODUCTION

FUZZY logic may be viewed as a generalization of multivalued logic in that it provides a wider range of tools for dealing with uncertainty and imprecision in knowledge representation, inference, and decision analysis. In particular, fuzzy logic allows the use of a) fuzzy predicates exemplified by *small*, *young*, *nice*, etc; b) fuzzy quantifiers exemplified by *most*, *several*, *many*, *few*, *many more*, etc; c) fuzzy truth values exemplified by *quite true*, *very true*, *mostly false*, etc. d) fuzzy probabilities exemplified by *likely*, *unlikely*, *not very likely* etc; e) fuzzy possibilities exemplified by *quite possible*, *almost impossible*, etc; and f) predicate modifiers exemplified by *very*, *more or less*, *quite*, *extremely*, etc.

What matters most about fuzzy logic is its ability to deal with fuzzy quantifiers as fuzzy numbers which may be manipulated through the use of fuzzy arithmetic. This ability depends in an essential way on the existence—within fuzzy logic—of the concept of cardinality or, more generally, the concept of measure of a fuzzy set. Thus if one accepts the classical view of Kolmogoroff that probability theory is a branch of measure theory, then, more generally, the theory of fuzzy probabilities may be subsumed within fuzzy logic. This aspect of fuzzy logic makes it particularly well-suited for the management of uncertainty in expert systems (Zadeh [48]). More specifically, by employing a single framework for the analysis of both probabilistic and possibilistic uncertainties, fuzzy logic provides a systematic basis for inference from premises which are imprecise, incomplete or not totally reliable. In this way it becomes possible, as is shown in this paper, to derive a set of rules for combining evidence through conjunction, disjunction, and chaining. In effect, such rules may be viewed as instances of syllogistic reasoning in fuzzy logic. However, unlike the rules employed in most of the existing expert systems, they are not *ad hoc* in nature.

Our concern in this paper is with fuzzy syllogisms of the general form

$$\begin{array}{c} p(Q_1) \\ \underline{q(Q_2)} \\ r(Q) \end{array} \tag{1.1}$$

in which the first premise $p(Q_1)$ is a fuzzy proposition containing a fuzzy quantifier Q_1; the second premise $q(Q_2)$ is a fuzzy proposition containing a fuzzy quantifier Q_2; and the conclusion $r(Q)$ is a fuzzy proposition containing a fuzzy quantifier Q. For example, the *intersection/product syllogism* may be expressed as

$$\begin{array}{c} Q_1 A\text{'s are } B\text{'s} \\ \underline{Q_2(A \text{ and } B)\text{'s are } C\text{'s}} \\ QA\text{'s are } (B \text{ and } C)\text{'s} \end{array} \tag{1.2}$$

where A, B, and C are labels of fuzzy sets, and the fuzzy quantifier Q is given by the product of the fuzzy quantifiers Q_1 and Q_2, i.e.

$$Q = Q_1 \otimes Q_2 \tag{1.3}$$

where ⊗ denotes the product in fuzzy arithmetic (Kaufmann and Gupta [21]) (Fig. 1).[1] It should be noted that (1.3) may be viewed as an analog of the basic probabilistic identity (Jaynes [19])

$$p(B, C/A) = p(B/A)p(C/A, B). \tag{1.4}$$

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[1] More generally, a circle around an arithmetic operator represents its extension to fuzzy operands.

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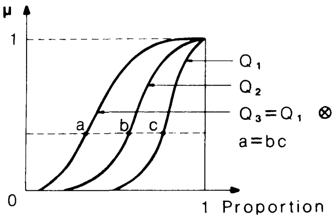

Fig. 1. Multiplication of fuzzy quantifiers.

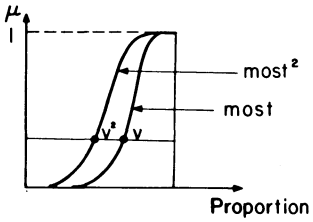

Fig. 2. Representation of *most* and *most*^{2}.

A concrete example of the intersection/product syllogism is the following

most students are young
most young students are single (1.5)
most^2 *students are young and single*

where *most*^{2} denotes the product of the fuzzy quantifier *most* with itself (Fig. 2).

For concreteness, we shall restrict our attention to syllogisms in which p, q, and r are propositions of the form

$$p \triangleq Q_1 A\text{'s are } B\text{'s} \tag{1.6}$$

$$q \triangleq Q_2 C\text{'s are } D\text{'s} \tag{1.7}$$

$$r \triangleq Q_3 E\text{'s are } F\text{'s} \tag{1.8}$$

in which A, B, C, D, E, and F are interrelated fuzzy predicates.

The interrelations between A, B, C, D, E, and F provide a basis for a classification of fuzzy syllogisms. The more important of these syllogisms are the following ($\wedge \triangleq$ denotes conjunction, and $\vee \triangleq$ denotes disjunction).

Intersection/product syllogism:

$$C = A \wedge B, \ E = A, \ F = C \wedge D. \tag{1.9}$$

Chaining syllogism:

$$C = B, \ E = A, \ F = D. \tag{1.10}$$

Consequent conjunction syllogism:

$$A = C = E, \ F = B \wedge D. \tag{1.11}$$

Consequent disjunction syllogism:

$$A = C = E, \ F = B \vee D. \tag{1.12}$$

Antecedent conjunction syllogism:

$$B = D = F, \ E = A \wedge C. \tag{1.13}$$

Antecedent disjunction syllogism:

$$B = D = F, \ E = A \vee C. \tag{1.14}$$

In the context of expert systems, these and related syllogisms provide a set of inference rules for combining evidence through conjunction, disjunction and chaining [48].

An important application of syllogistic reasoning in fuzzy logic relates to what may be regarded as reasoning with *dispositions* [50]. A disposition, as its name suggests, is a proposition which is preponderantly but not necessarily always true. To capture this intuitive meaning of a disposition, we define a disposition as a proposition with implicit extremal fuzzy quantifiers, e.g., *most*, *almost all*, *almost always*, *usually*, *rarely*, *few*, *small fraction*. etc. This definition should be regarded as a *dispositional definition* in the sense that it may not be true in all cases.

Examples of commonplace statements of fact which may be viewed as dispositions are *overeating causes obesity*, *snow is white*, *glue is sticky*, *icy roads are slippery*, etc. An example of what appears to be a plausible conclusion drawn from dispositional premises is the following

icy roads are slippery
slippery roads are dangerous (1.15)
icy roads are dangerous.

As will be seen in Section III, syllogistic reasoning with dispositions provides a basis for a formalization of the type of commonsense reasoning exemplified by (1.15).

The importance of the concept of a disposition stems from the fact that what is commonly regarded as commonsense knowledge may be viewed as a collection of dispositions [49]. For example, the dispositions

birds can fly
small cars are unsafe
professors are not rich
students are young
where there is smoke there is fire
Swedes are taller than Italians
Swedes are blond
it takes a little over one hour to drive
from Berkeley to Stanford
a TV set weighs about 50 *pounds*

may be regarded as a part of commonsense knowledge. The last two examples typify a particularly important type of disposition: *a dispositional valuation*, that is, a disposition which characterizes the usual value of a variable through the implicit fuzzy quantifier usually. Thus upon *explicitation*, that is, making explicit the implicit fuzzy quantifiers, the examples in question become

usually it takes a little over one hour to drive
from Berkeley to Stanford
usually a TV set weighs about 50 *pounds*.

Dispositional valuations have the general form

$$(\textit{usually})(X \text{ is } F) \tag{1.16}$$

where (*usually*) is an implicit fuzzy quantifier; X is a variable which is constrained by the disposition; and F is its fuzzy value. As an illustration, expressed in this form, the last example reads[2]

$$(\textit{usually})(\textit{weight}(TV) \text{ is ABOUT.FIFTY.POUNDS}).$$

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[2] Upper case symbols and periods serve to represent compound labels of fuzzy sets.

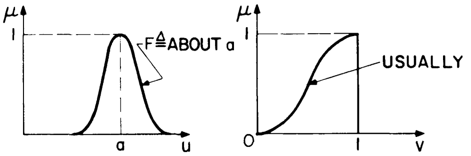

Fig. 3. Representation of (*usually*)(X is ABOUT a).

A basic syllogism which governs inference from dispositional valuations is the *dispositional modus ponens*

$$\begin{array}{l} (\textit{usually})(X \text{ is } F) \\ \underline{\text{if } X \text{ is } F \text{ then } (\textit{usually})(Y \text{ is } G)} \\ (\textit{usually}^2)(Y \text{ is } G), \end{array} \tag{1.17}$$

where F and G are fuzzy predicates and *usually*^{2} is the product of the fuzzy quantifier *usually* with itself.

To a first approximation, the fuzzy quantifier *usually* may be represented as a fuzzy number of the same form as *most* (Fig. 3). More generally, however, *usually* connotes a dependence on the assumption of "normality." More specifically, let Z denote what might be called a conditioning variable whose normal (or regular) value is R, where R in general is a fuzzy set which is the complement of a set of exceptions. In terms of the conditioning variable, the dispositional valuation (*usually*)(X is F) may be expressed as

$$(\textit{usually})(X \text{ is } F) \leftrightarrow \text{if } (Z \text{ is } R) \quad \text{then } (\textit{most } X\text{'s are } F). \tag{1.18}$$

For example

$$\begin{array}{l} (\textit{usually})(\textit{duration}(\textit{trip}(\textit{Berkeley}, \textit{ Stanford})) \text{ is} \\ \text{LITTLE.OVER.ONE. HOUR}) \leftrightarrow \text{if } ((\textit{time}(\textit{trip})) \text{ is} \\ \text{NOT.RUSH.HOUR}) \text{ then } (\textit{most durations}(\textit{trip} \\ (\textit{Berkeley}, \textit{ Stanford}) \text{ are LITTLE.OVER.ONE.HOUR}. \end{array}$$

What these examples are intended to suggest is that the concept of a disposition plays an essential role in the representation and inference from commonsense knowledge. A more detailed discussion of this role of dispositions will be presented in Section III.

II. FUZZY QUANTIFIERS, COMPOSITIONALITY, ROBUSTNESS, AND USUALITY

As was stated in the introduction, the concept of a fuzzy quantifier is related in an essential way to the concept of cardinality—or, more, generally, the concept of measure—of fuzzy sets. More specifically, a fuzzy quantifier may be viewed as a fuzzy characterization of the absolute or relative cardinality of a collection of fuzzy sets. In this sense, then, a fuzzy quantifier is a second-order fuzzy predicate.

The cardinality of a fuzzy set may be defined in a variety of ways [47]. For simplicity, we shall employ the *sigma-count* for this purpose, which is defined as follows [6], [45].

Let A be a finite fuzzy subset of the universe of discourse U, with A expressed as

$$A = \mu_1/u_1 + \cdots + \mu_n/u_n \tag{2.1}$$

where μ_1/u_1, $i = 1, \cdots, n$, signifies that μ_i is the grade of membership of u_i in A and $+$ denotes the union. Then, the sigma-count of A is defined as the real number

$$\Sigma\text{count}(A) = \Sigma_i \mu_i \tag{2.2}$$

with the understanding that the sum may be rounded to the nearest integer, if necessary. Furthermore, one may stipulate that the terms whose grade of membership fall below a specified threshold be excluded from the summation. The purpose of such an exclusion is to avoid a situation in which a large number of terms with low grades of membership become count-equivalent to a small number of terms with high membership.

The relative sigma-count, denoted by Σcount (B/A), may be interpreted as the proportion of elements of B in A. More explicitly,

$$\Sigma\text{count}(B/A) = \frac{\Sigma\text{count}(B \cap A)}{\Sigma\text{count}(A)} \tag{2.3}$$

where $B \cap A$, the intersection of B and A, is defined by

$$\mu_{B \cap A}(u) = \mu_B(u) \wedge \mu_A(u), \qquad u \in U. \tag{2.4}$$

Thus, in terms of the membership functions of B and A, the relative sigma-count of B in A is given by

$$\Sigma \text{ count}(B/A) = \frac{\Sigma_i \mu_B(u_i) \wedge \mu_A(u_i)}{\Sigma_i \mu_A(u_i)}. \tag{2.5}$$

The concept of a relative sigma-count provides a basis for interpreting the meaning of propositions of the form $p \triangleq QA$'s are B's, e.g., *most young men are healthy*. More specifically, the fuzzy quantifier Q in the proposition QA's are B's may be regarded as a fuzzy characterization of the relative sigma-count of B in A, which entails that the proposition in question may be translated as

$$QA\text{'s are } B\text{'s} \rightarrow \Sigma\text{count}(B/A) \text{ is } Q. \tag{2.6}$$

The right-hand member of (2.6) implies that Q, viewed as a fuzzy number, defines the possibility distribution of Σ count(B/A). This may be expressed as the *possibility assignment equation* [45]

$$\Pi_X = Q \tag{2.7}$$

in which the variable X is the sigma-count in question and Π_X is its possibility distribution.

As was stated earlier, a fuzzy quantifier is a second-order fuzzy predicate. The interpretation expressed by (2.6) and (2.7) shows that the evaluation of a fuzzy quantifier may be reduced to that of a first order predicate if Q is interpreted as a fuzzy subset of the real line. Thus let us consider again the proposition $p \triangleq QA$'s are B's, in which A and B are fuzzy sets in their respective universes of discourse, U and V; and Q, regarded as a second-order fuzzy predicate, is assumed to be characterized by its membership function $\mu_Q(X, Y)$, with X and Y ranging over the fuzzy subsets of U and V. Then based on (2.6) and (2.7), we can define $\mu_Q(X, Y)$ through the equality.

$$\mu_Q(X, Y) = \mu_Q(\Sigma\text{count}(X/Y)) \tag{2.8}$$

in the right-hand member of which Q is a unary first-order fuzzy predicate whose denotation is a fuzzy subset of the unit interval. Consequently, in the proposition QA's are B's, Q may be interpreted as a) a second-order fuzzy predicate defined on $U^* \times V^*$, where U^* and V^* are the fuzzy power sets of U and V or b) a first-order fuzzy predicate defined on the unit interval $[0, 1]$.

It is useful to classify fuzzy quantifiers into quantifiers of the first kind, second kind, third kind, etc., depending on the arity of the second-order fuzzy predicate which the quantifier represents. Thus, Q is a fuzzy quantifier of the first kind if it provides a fuzzy characterization of the cardinality of a fuzzy set; Q is of the second kind if it provides a fuzzy characterization of the relative cardinality of two fuzzy sets; and Q is of the third kind if it serves the same role in relation to three fuzzy sets. For example, the fuzzy quantifier labeled *several* is of the first kind; *most* is of the second kind; and *many more* in *there are many more A's in B's than A's in C's* is of the third kind. It should be noted that, in terms of this classification, the certainty factors employed in such expert systems as MYCIN [38] and PROSPECTOR [9] are fuzzy quantifiers of the third kind.

The concept of a fuzzy quantifier gives rise to a number of other basic concepts relating to syllogistic reasoning, among which are the concepts of *compositionality* and *robustness*.

Specifically, consider a fuzzy syllogism of the general form (1.1), i.e.

$$\begin{array}{c} p(Q_1) \\ q(Q_2) \\ \hline r(Q). \end{array} \tag{2.9}$$

We shall say that the syllogism is strongly compositional if a) Q may be expressed as a function of Q_1 and Q_2 independent of the denotations of the predicates which enter into p and q, excluding the trivial case, where Q is the unit interval, and b) if Q_1 and Q_2 are numerical quantifiers, so is Q. Furthermore, we shall say that the syllogism is weakly compositional if only a) is satisfied, in which case if Q_1 and Q_2 are numerical quantifiers, Q may be interval-valued. As will be seen in the sequel, in order to achieve strict compositionality, it is necessary, in general, to make some restrictive assumptions concerning the predicates in p and q. For example, the syllogism

$$\begin{array}{l} Q_1A\text{'s are }B\text{'s} \\ Q_2B\text{'s are }C\text{'s} \\ \hline (Q_1 \otimes Q_2)A\text{'s are }(B\text{ and }C)\text{'s} \end{array} \tag{2.10}$$

is strictly compositional if $B \subset A$.

Turning to the concept of robustness, suppose that we start with a nonfuzzy syllogism of the form

$$\begin{array}{c} p(\text{all}) \\ q(\text{all}) \\ \hline r(\text{all}) \end{array} \tag{2.11}$$

an example of which is

$$\begin{array}{c} \text{all }A\text{'s are }B\text{'s} \\ \text{all }B\text{'s are }C\text{'s} \\ \hline \text{all }A\text{'s are }C\text{'s.} \end{array} \tag{2.12}$$

The original syllogism is *robust* if small perturbations in the quantifiers in p and q result in a small perturbation in the quantifier in r. For example, the syllogism represented by (2.12) is robust if its validity is preserved when a) the quantifier *all* in p and q is replaced by *almost all*, and b) the quantifier *all* in r is replaced by *almost almost all*. (In more concrete terms, this is equivalent to replacing *all* in p and q by the fuzzy number $1 \ominus \epsilon$, where ϵ is a small fuzzy number and b) replacing *all* in r by the fuzzy number $1 \ominus \epsilon$.) More generally, a syllogism is selectively robust if the above holds for perturbations in either the first or the second premise, but not necessarily in both. For example, it may be shown that the syllogism expressed by (2.12) is selectively robust with respect to perturbations in the first premise but not in the second premise. In fact, the syllogism in question is brittle with respect to perturbations in the second premise in the sense that the slightest perturbation in the quantifier *all* in q requires the replacement of the quantifier *all* in r by the vacuous quantifier *none-to-all*.

As was stated earlier, the concept of dispositionality is closely related to that of usuality. In particular, a dispositional valuation expressed as X is F may be interpreted as (*usually*)(X is F) or equivalently as $U(X) = F$, where *usually* is an implicit fuzzy quantifier, and $U(X)$ denotes the usual value of X. For example, *glue is sticky* may be interpreted as (*usually*)(*glue is sticky*); *lamb is more expensive than beef* may be interpreted as (*usually*)(*lamb is more expensive than beef*); and *most students are undergraduates* may be interpreted in some contexts as (*usually*)(*most students are undergraduates*).

To concretize the meaning of the fuzzy quantifier *usually*, assume that X takes a sequence of values $X_1, \cdots, X_n$ in a universe of discourse U. Then, *usually*, in its unconditioned sense, may be defined by

$$\textit{usually}\ (X \text{ is } F) \triangleq \textit{most}\ X\text{'s are } F \tag{2.13}$$

where F plays the role of a usual value $U(X)$ of X. An immediate consequence of this definition is that a usual value of X is not unique. Rather, given a fuzzy value F, we can compute the degree τ to which it satisfies the definition (2.13) by employing the formula

$$\tau = \mu_{\text{MOST}}\left[\frac{1}{n}(\mu_F(X_1) + \cdots + \mu_F(X_n))\right] \tag{2.14}$$

where μ_{MOST} is the membership function of the fuzzy quantifier most; μ_F is the membership function of F; and the argument of μ_{MOST} is the relative sigma-count of the number of times X satisfies the proposition X is F.

More generally, when *usually* is defined in its conditioned sense via (1.18), the degree of compatibility of F with the definition of a usual value may be expressed as

$$\tau = \mu_{\text{MOST}}(\Sigma\text{count}(F/R))$$

in which the relative sigma-count is given by

$$\Sigma\text{count}(F/R) = \frac{\Sigma_i \mu_F(X_i) \wedge \mu_R(X_i)}{\Sigma_i \mu_R(X_i)}, \qquad i = 1,\ldots,n.$$

The importance of the concept of usuality stems from the fact that it underlies almost all of human decisionmaking from the time of awakening in the morning till retirement at night. Thus in deciding on when to get up, we take into consideration how long it usually takes to dress, have breakfast, drive to work, etc. In fact, we could not function at all without a knowledge of the usual values of the variables which enter into our daily decisionmaking and an ability to employ them in decision analysis. It is of interest to observe that the usual values are usually fuzzy rather than crisp. How to manipulate the usual values in the context of fuzzy syllogisms will be discussed briefly in the following section.

III. Fuzzy Syllogisms and Reasoning with Dispositions

As was stated earlier, one of the basic syllogisms in fuzzy logic is the intersection/product syllogism expressed by (1.2).

In what follows, we shall employ this syllogism as a starting point for the derivation of other syllogisms which are of relevance to the important problem of combination of evidence in expert systems.

A derivative syllogism of this type is the multiplicative chaining syllogism

$$\begin{array}{l} Q_1 A\text{'s are } B\text{'s} \\ \underline{Q_2 B\text{'s are } C\text{'s}\qquad\qquad} \\ \geqslant (Q_1 \otimes Q_2) A\text{'s are } C\text{'s} \end{array} \tag{3.1}$$

in which $\geqslant (Q_1 \otimes Q_2)$ should be read as at least $Q_1 \otimes Q_2$. This syllogism is a special case of the intersection/product syllogism which results when $B \subset A$, i.e.

$$\mu_B(u_i) < \mu_A(u_i), \qquad u_i \in U,\ i = 1, \cdots . \tag{3.2}$$

For, in this case $A \cap B \equiv B$, and since $B \cap C$ is contained in C, it follows that

$$(Q_1 \otimes Q_2) A\text{'s are } (B \text{ and } C)\text{'s} \Rightarrow$$
$$\geqslant (Q_1 \otimes Q_2) A\text{'s are } C\text{'s}. \tag{3.3}$$

(It is of interest to note that if Q in the proposition QA's are B's is interpreted as the degree to which A is contained in B, then the multiplicative chaining syllogism shows that, under the assumption $B \subset A$, the fuzzy relation of fuzzy-set containment is *product transitive* [43], [48].

If, in addition to assuming that $B \subset A$, we assume that Q_1 and Q_2 are monotone increasing [3], [47], i.e.

$$\begin{array}{l} \geqslant Q_1 = Q_1 \\ \geqslant Q_2 = Q_2, \end{array} \tag{3.4}$$

which is true of the fuzzy quantifier *most*, then

$$\geqslant (Q_1 \otimes Q_2) = Q_1 \otimes Q_2 \tag{3.5}$$

and the multiplicative chaining syllogism becomes (Fig. 5)

$$\begin{array}{l} Q_1 A\text{'s are } B\text{'s} \\ \underline{Q_2 B\text{'s are } C\text{'s}\qquad\qquad} \\ (Q_1 \otimes Q_2) A\text{'s are } C\text{'s}. \end{array} \tag{3.6}$$

As an illustration, we shall consider an example in which the containment relation $B \subset A$ holds approximately, as in the proposition

$$p \triangleq \textit{most American cars are big}. \tag{3.7}$$

Then, if

$$q \triangleq \textit{most big cars are expensive} \tag{3.8}$$

we may conclude, by employing (3.6), that

$$r \triangleq \textit{most}^2 \textit{ American cars are expensive}$$

with the understanding that $most^2$ is the product of the fuzzy number *most* with itself [47].

It can readily be shown by examples that if no assumptions are made regarding A, B, and C, then the chaining inference schema

$$\begin{array}{l} Q_1 A\text{'s are } B\text{'s} \\ \underline{Q_2 B\text{'s are } C\text{'s}} \\ QA\text{'s are } C\text{'s}. \end{array} \tag{3.9}$$

is not weakly compositional, which is equivalent to saying that, in general, Q is the vacuous quantifier *none-to-all*. However, if we assume, as done above, that $B \subset A$, then it follows from the intersection/product syllogism that (3.6) becomes weakly compositional, with

$$Q = \geqslant (Q_1 \otimes Q_2) \tag{3.10}$$

and, furthermore, that (3.6) becomes strongly compositional if Q_1 and Q_2 are monotone increasing.

Another important observation relates to the robustness of the multiplicative chaining syllogism. Specifically, if we assume that

$$Q_1 = 1 \ominus \epsilon_1$$
$$Q_2 = 1 \ominus \epsilon_2$$

where ϵ_1 and ϵ_2 are small fuzzy numbers, then it can readily be verified that, approximately

$$Q_1 \otimes Q_2 \cong 1 \ominus \epsilon_1 \ominus \epsilon_2 \tag{3.11}$$

which establishes that the multiplicative chaining syllogism is robust. However, in the absence of the assumption $B \subset A$, the inference schema (3.9) is robust only with respect to perturbations in Q_1. To demonstrate this, assume that Q_1 = *almost all* and Q_2 = *all*. Then, from the intersection/product syllogism it follows that $Q = \geqslant$ (*almost all*). On the other hand, if we assume that Q_1 = *all* and Q_2 = *almost all*, then Q = *none-to-all*. Thus, as was stated earlier, the inference schema (3.9) is brittle with respect to perturbations in the second premise.

The MPR Chaining Syllogism

In the preceding discussion, we have shown that the assumption $B \subset A$ leads to a weakly compositional multiplicative chaining syllogism. Another type of assumption which also leads to a weakly compositional syllogism is that of major premise reversibility (MPR).[3] This assumption may be expressed as the semantic equivalence

$$Q_1 A\text{'s are } B\text{'s} \leftrightarrow Q_1 B\text{'s are } A\text{'s} \tag{3.12}$$

which, in most cases, will hold approximately rather than exactly. For example,

most American cars are big ↔ *most big cars are American.*

It can be shown (Zadeh [49]) that under the assumption of reversibility the following chaining syllogism holds in an approximate sense

$$\begin{array}{l} Q_1 A\text{'s are } B\text{'s} \\ \underline{Q_2 B\text{'s are } C\text{'s}\qquad\qquad\qquad} \\ \geqslant \big(0 \ovee ((Q_1 \oplus Q_2 \ominus 1))A\text{'s are } C\text{'s.} \end{array} \tag{3.13}$$

We shall refer to this syllogism as the *MPR chaining syllogism*. It follows at once from (3.13) that the MPR chaining syllogism is weakly compositional and robust. A concrete instance of this syllogism is provided by the following example

$$\begin{array}{l} \textit{most American cars are big} \\ \underline{\textit{most big cars are heavy}\qquad\qquad} \\ 0 \ovee (2\ \textit{most} \ominus 1)\ \textit{American cars are heavy.} \end{array} \tag{3.14}$$

The Consequent Conjunction Syllogism

The consequent conjunction syllogism is an example of a basic syllogism which is not a derivative of the intersection/product syllogism. Its statement may be expressed as follows:

$$\begin{array}{l} Q_1 A\text{'s are } B\text{'s} \\ \underline{Q_2 A\text{'s are } C\text{'s}\qquad} \\ QA\text{'s are } (B \text{ and } C)\text{'s} \end{array} \tag{3.15}$$

where

$$0 \ovee (Q_1 \oplus Q_2 \ominus 1) < Q < Q_1 \owedge Q_2. \tag{3.16}$$

From (3.16), it follows at once that the syllogism is weakly compositional and robust.

An illustration of (3.15) is provided by the example

$$\begin{array}{l} \textit{most students are young} \\ \underline{\textit{most students are single}\qquad} \\ \textit{Q students are single and young} \end{array}$$

where

$$2\text{most} \ominus 1 \leqslant Q \leqslant \text{most}. \tag{3.17}$$

This expression for Q follows from (3.16) by noting that

$$\text{most} \owedge \text{most} = \text{most}$$

and

$$0 \ovee (2\text{most} \ominus 1) = 2\text{most} \ominus 1.$$

The importance of the consequent conjunction syllogism stems from the fact that it provides a formal basis for combining rules in an expert system through a conjunctive combination of hypotheses [38]. However, unlike such rules in MYCIN and PROSPECTOR, the consequent conjunction syllogism is weakly rather than strongly compositional. Since the combining rules in MYCIN and PROSPECTOR are *ad hoc* in nature, whereas the consequent conjunction syllogism is not, the validity of strong compositionality in MYCIN and PROSPECTOR is in need of justification.

The Antecedent Conjunction Syllogism

An issue which plays an important role in the management of uncertainty in expert systems relates to the question of how to combine rules which have the same consequent but different antecedents.

Expressed as an inference schema in fuzzy logic, the question may be stated as

$$\begin{array}{l} Q_1 A\text{'s are } C\text{'s} \\ \underline{Q_2 B\text{'s are } C\text{'s}\qquad} \\ Q(A \text{ and } B)\text{'s are } C\text{'s} \end{array} \tag{3.18}$$

in which Q is the quantifier to be determined as a function of Q_1 and Q_2.

It can readily be shown by examples that, in the absence of any assumptions about A, B, C, Q_1, and Q_2, what can be said about Q is that it is the vacuous quantifier *none-to-all*. Thus, to be able to say more, it is necessary to make some restrictive assumptions which are satisfied, at least approximately, in typical situations.

The commonly made assumption in the case of expert systems [9], is that the items of evidence are conditionally independent given the hypothesis. Expressed in terms of the relative sigma-counts of A, B, and C, this assumption may be written as

$$\Sigma\text{count}(A \cap B/C) = \Sigma\text{count}(A/C)\Sigma\text{count}(B/C). \tag{3.19}$$

Using this equality, it is easy to show that

$$\Sigma\text{count}(C/A \cap B) = K\Sigma\text{count}(C/A)\Sigma\text{count}(C/B) \tag{3.20}$$

where the factor K is given by

$$K = \frac{\Sigma\text{count}(A)\ \Sigma\text{count}(B)}{\Sigma\text{count}(A \cap B)\Sigma\text{count}(C)}. \tag{3.21}$$

The presence of this factor has the effect of making the inference schema (3.18) noncompositional. One way of getting around the problem is to employ—instead of the sigma-count—a count defined by

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[3] It should be noted that, in the classical theory of syllogisms, the premise in question would be referred to as the minor premise.

$$\rho\Sigma\text{count}(B) = \frac{\Sigma\text{count}(B)}{\Sigma\text{count}(\neg B)} \tag{3.22}$$

$$\rho\Sigma\text{count}(B/A) = \frac{\Sigma\text{count}(B/A)}{\Sigma\text{count}(\neg B/A)} \tag{3.23}$$

in which $\neg B$ denotes the negation of B (or, equivalently, the complement of B, if B is interpreted as a fuzzy set which represents the denotation of the predicate B). These counts will be referred to as *ρsigma-counts* (with ρ standing for ratio) and correspond to the odds which are employed in PROSPECTOR. Thus, expressed in words, we have

$$\rho\Sigma\text{count}(B) \triangleq \text{ratio of } B\text{'s } \textit{to} \text{ non-}B\text{'s} \tag{3.24}$$

$$\rho\Sigma\text{count}(B/A) \triangleq \text{ratio of } B\text{'s } \textit{to} \text{ non-}B\text{'s among } A\text{'s.} \tag{3.25}$$

In terms of ρsigma-counts, it can readily be shown that the assumption expressed by (3.19) entails the equality

$$\rho\Sigma\text{count}(C/A \cap B) = \rho\Sigma\text{count}(C/A)\rho\Sigma\text{count}(C/B) \cdot \rho\Sigma\text{count}(\neg C). \tag{3.26}$$

This equality, then, leads to what will be referred to as the *antecedent conjunction syllogism*

$$\begin{array}{l} \text{ratio of } C\text{'s to non-}C\text{'s among } A\text{'s is } R_1 \\ \underline{\text{ratio of } C\text{'s } \textit{to} \text{ non-}C\text{'s among } B\text{'s is } R_2} \\ \text{ratio of } C\text{'s } \textit{to} \text{ non-}C\text{'s} \\ \quad \text{among } (A \text{ and } B)\text{'s is } R_1 \otimes R_2 \otimes R_3 \end{array} \tag{3.27}$$

where

$$R_3 \triangleq \text{ratio of } C\text{'s to non-}C\text{'s.}$$

It should be noted that this syllogism may be viewed as the fuzzy logic analog of the likelihood ratio combining rule in PROSPECTOR [9].

In the foregoing discussion, we have focused our attention on some of the basic syllogisms in fuzzy logic which may be employed as rules of combination of evidence in expert systems. Another important function which these and related syllogisms may serve is that of providing a basis for reasoning with dispositions, that is, with propositions in which there are implicit fuzzy quantifiers.

The basic idea underlying this application of fuzzy syllogisms is the following. Suppose that we are given two dispositions

$$\begin{array}{l} \textit{icy roads are slippery} \\ \textit{slippery roads are dangerous.} \end{array} \tag{3.28}$$

Can we infer from these dispositions what appears to be a plausible conclusion, namely

icy roads are dangerous?

As a first step, we have to restore the suppressed fuzzy quantifiers in the premises. For simplicity, assume that the desired restoration may be accomplished by prefixing the dispositions in question with the fuzzy quantifier most, i.e.

icy roads are slippery → *most icy roads are slippery*

slippery roads are dangerous

→ *most slippery roads are dangerous.*

Next, if we assume that the proposition *most slippery roads are dangerous* satisfies the major premise reversibility condition, i.e.,

most icy roads are slippery

↔ *most slippery roads are icy*

then by applying the MPR chaining syllogism (3.13) we have

$$\begin{array}{l} \textit{most icy roads are slippery} \\ \underline{\textit{most slippery roads are dangerous,}} \\ (2\text{most} \ominus 1)\ \textit{icy roads are dangerous.} \end{array} \tag{3.29}$$

Finally, on suppressing the fuzzy quantifiers in (3.29), we are led to the chain of dispositions

$$\begin{array}{l} \textit{icy roads are slippery} \\ \underline{\textit{slippery roads are dangerous}} \\ \textit{icy roads are dangerous} \end{array} \tag{3.30}$$

which answers in the affirmative the question posed in (3.28), with the understanding that the implicit fuzzy quantifier in the conclusion of (3.28) is *2most ⊖ 1* rather than *most*.

In a more general setting, reasoning with dispositions may be viewed as a part of what might be called a *theory of usuality*. In earlier sections, we have already defined the concept of a usual value and stated a rule of inference which was referred to as the *dispositional modus ponens*. In what follows, we shall sketch a few additional rules of this nature which involve the concept of usuality. A more detailed account of the theory of usuality and its applications to commonsense reasoning will be presented in a forthcoming paper.

The rules described below have one basic characteristic in common, namely, they are dispositional variants of categorical (ie., nondispositional) rules of inference in fuzzy logic. In other words, such rules may be viewed as the result of replacing one or more categorical premises, say $p_1, \cdots, p_k$, with dispositional premises expressed as $(\textit{usually})p_1, (\textit{usually})p_2, \cdots, (\textit{usually})p_k$.

As a simple illustration, a basic inference rule in fuzzy logic is the *entailment principle*, which may be expressed as

$$\begin{array}{c} X \text{ is } A \\ \underline{A \subset B} \\ X \text{ is } B \end{array} \tag{3.31}$$

where A and B are fuzzy predicates. In plain words, this rule states that any assertion about X of the form X is A entails any less specific assertion X is B.

A dispositional variant of the entailment principle may be expressed as

$$\begin{array}{l} (\textit{usually})(X \text{ is } A) \\ \underline{A \subset B \qquad\qquad\quad} \\ (\textit{usually})(X \text{ is } B). \end{array} \tag{3.32}$$

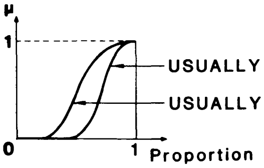

Fig. 4. Representation of *usually* and $usually^2$.

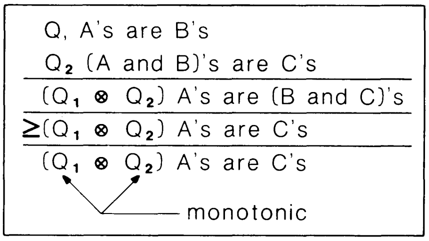

Fig. 5. Representation of the intersection/product syllogism and its corollaries.

Another variant is

$$\frac{\begin{array}{c} X \text{ is } A \\ (usually)(A \subset B) \end{array}}{(usually)(X \text{ is } B).} \tag{3.33}$$

Still another variant is

$$\frac{\begin{array}{c} (usually)(X \text{ is } A) \\ (usually)(A \subset B) \end{array}}{(usually^2)(X \text{ is } B).} \tag{3.34}$$

Note that $usually^2$ is less specific than *usually* (Fig. 4).

A special case of the entailment principle which plays an important role in fuzzy logic is the *projection rule*, which may be stated as follows. Let X and Y be variables ranging over U and V, respectively, and let R be a fuzzy relation in $U \times V$, which is characterized by its membership function $\mu_R(u, v)$, $u \in U$, $v \in V$. Then, in symbols, the projection rule may be expressed as

$$\frac{(X, Y) \text{ is } R}{X \text{ is } \mathrm{proj}_U\, R} \tag{3.35}$$

where the first premise signifies that X and Y are R-related, and $\mathrm{proj}_U\, R$ denotes the projection of R on U. The membership function of $\mathrm{proj}_U\, R$ is given by

$$\mu_{\mathrm{Proj}_U} R(u) = \sup_v \mu_R(u, v). \tag{3.36}$$

Since R is contained in $\mathrm{proj}_U\, R$, it follows from (3.32) that

$$\frac{(usually)((X, Y) \text{ is } R)}{(usually)(X \text{ is } \mathrm{Proj}_U\, R).} \tag{3.37}$$

This inference rule may be regarded as a dispositional version of the projection rule.

The dispositional inference rule expressed by (3.37) may be applied to the derivation of a dispositional form of the compositional rule of inference in fuzzy logic [45]

$$\frac{\begin{array}{c} X \text{ is } A \\ (X, Y) \text{ is } R \end{array}}{Y \text{ is } A \cdot R} \tag{3.38}$$

where $A \cdot R$ denotes the composition of A and R, which is defined by

$$\mu_{A \cdot R}(v) = \sup_u (\mu_A(u) \wedge \mu_R(u, v)). \tag{3.39}$$

Specifically, assume that the premises in (3.38) are expressed as the dispositional valuations

$$(usually)(X \text{ is } A) \tag{3.40}$$

and

$$(usually)((X, Y) \text{ is } R) \tag{3.41}$$

in which the fuzzy quantifier *usually* is the same in the two premises in the sense that the conjunction of the premises in question may be expressed as

$$(usually)((X \text{ is } A) \wedge ((X, Y) \text{ is } R))$$

or, equivalently, as

$$(usually)(X, Y) \text{ is } A \wedge R) \tag{3.42}$$

where the conjunction of A and R is defined by

$$\mu_{A \wedge R}(u, v) = \mu_A(u) \wedge \mu_R(u, v). \tag{3.43}$$

Then, on applying the dispositional projection rule to (3.41), we have

$$\frac{(usually)((X \text{ is } A) \wedge ((X, Y) \text{ is } R))}{(usually)(Y \text{ is } \mathrm{proj}_V\, (A \wedge R)} \tag{3.44}$$

and since

$$A \cdot R = \mathrm{proj}_V(A \wedge R)$$

we can assert that

$$\frac{\begin{array}{c} (usually)(X \text{ is } A) \\ (usually)((X, Y) \text{ is } R) \end{array}}{(usually)(Y \text{ is } A \cdot R),} \tag{3.45}$$

which expresses a dispositional version of the compositional rule of inference. It should be noted that the validity of this version depends on the assumption that the two premises (3.40) and (3.41) may be combined conjunctively as in (3.42). Without this assumption, the fuzzy quantifier in the conclusion would be less specific than *usually*, e.g., it might be $usually^2$.

An important way in which usuality enters into syllogistic reasoning is the following.

Consider the chaining inference schema

$$\frac{\begin{array}{c} Q_1 A\text{'s are } B\text{'s} \\ Q_2 B\text{'s are } C\text{'s} \end{array}}{QA\text{'s are } C\text{'s}} \tag{3.46}$$

in which no restrictions are placed on A, B, C. Then, as was pointed out earlier (see (3.9)), Q is the vacuous quantifier *none-to-all*. However, under the assumption that $B \subset A$, (3.46) leads to the multiplicative chaining syllogism (see (3.1))

$$\begin{array}{l} Q_1 A\text{'s are } B\text{'s} \\ Q_2 B\text{'s are } C\text{'s} \\ \underline{B \subset A \qquad\qquad} \\ \geqslant (Q_1 \otimes Q_2) A\text{'s are } C\text{'s.} \end{array} \tag{3.47}$$

Now assume that the categorical assumption $B \subset A$ is replaced by the dispositional assumption

$$(usually)(B \subset A). \tag{3.48}$$

With this assumption, (3.45) becomes what we shall refer to as the multiplicative chaining *hypersyllogism*

$$\begin{array}{l} Q_1 A\text{'s are } B\text{'s} \\ Q_2 B\text{'s are } C\text{'s} \\ \underline{(usually)(B \subset A) \qquad\qquad} \\ (usually)(\geqslant (Q_1 \otimes Q_2) A\text{'s are } C\text{'s}) \end{array} \tag{3.49}$$

with the understanding that the prefix *hyper* signifies that the fuzzy quantifier *usually* in application to the containment relation $B \subset A$ is a higher-order fuzzy quantifier.

CONCLUSION

Syllogistic reasoning in fuzzy logic is a basic mode of inference which provides a) a basis for a formalization of commonsense reasoning and b) a non *ad hoc* computational framework for combining evidence in expert systems. In the latter application, the specificity of conclusions may be sharpened by utilizing higher-order dispositional assumptions regarding the predicates that enter into the premises. In this way, we are led to the concept of *hypersyllogistic reasoning*—a mode of reasoning which may eventually play an important role in the management of uncertainty in knowledge-based systems.

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REFERENCES

[1] E. W. Adams and H. F. Levine, "On the uncertainties transmitted from premises to conclusions in deductive inferences," *Synthese*, vol. 30, pp. 429–460, 1975.

[2] J. F. Baldwin and S. Q. Zhou, "A fuzzy relational inference language," rep. no. EM/FS132, University of Bristol, England, 1982.

[3] J. Barwise and R. Cooper, "Generalized quantifiers and natural language," *Linguistics and Philosophy*, vol. 4, pp. 159–219, 1981.

[4] R. E. Bellman and L. A. Zadeh, "Local and fuzzy logics," in *Modern Uses of Multiple-Valued Logic*, G. Epstein, Ed. Dordrecht: Reidel, 1977, pp. 103–165.

[5] S. Cushing, *Quantifier Meanings—A Study in the Dimensions of Semantic Competence*. Amsterdam: North-Holland, 1982.

[6] A. DeLuca and S. Termini, "A definition of non-probabilistic entropy in the setting of fuzzy sets theory," *Info. Control*, vol. 20, pp. 301–312, 1972.

[7] D. Dubois and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*. New York: Academic, 1980.

[8] ——, *Theorie des Possibilites*. Paris: Masson, 1985.

[9] R. O. Duda, P. E. Hart, K. Konolige, and R. Reboh, "A computer-based consultant for mineral exploration," tech. rep., SRI International, Menlo Park, CA, 1979.

[10] P. S. Ekberg and L. Lopes, "Tests of a natural reasoning model for syllogistic reasoning," *Goteborg Psychological Reports*, University of Goteborg, Sweden, vol. 9, no. 3, 1979.

[11] J. M. Francioni, A. Kandel, and C. M. Eastman, "Imprecise decision tables and their optimization," in *Approximate Reasoning in Decision Analysis*, M. Gupta, and E. Sanchez, Eds. Amsterdam: North-Holland, pp. 89–96, 1982.

[12] B. R. Gaines, "Precise past—fuzzy future," *Int. J. of Man-Machine Studies*, vol. 19, pp. 117–134.

[13] J. A. Goguen, "The logic of inexact concepts," *Synthese*, vol. 19, pp. 325–373, 1969.

[14] I. Hacking, *Logic of Statistical Inference*. Cambridge: Cambridge University, 1965.

[15] C. G. Hempel, "Maximal specificity and lawlikeness," *Philosophy of Science*, vol. 35, pp. 116–133, 1968.

[16] J. Hintikka, "Statistics, induction and lawlikeness," *Synthese*, vol. 20, pp. 72–83, 1969.

[17] D. N. Hoover, "Probability logic," *Annals Math. Logic*, vol. 14, pp. 287–313, 1978.

[18] M. Ishizuka, K. S. Fu, and J. T. P. Yao, "A rule-based inference with fuzzy set for structural damage assessment," in *Fuzzy Information and Decision Processes*, M. Gupta, and E. Sanchez, Eds. Amsterdam: North-Holland, 1982.

[19] E. T. Jaynes, "New engineering applications of information theory," in *Proc. First Symp. Eng. Applications of Random Function Theory and Probability*, J. L. Bogdanoff and F. Kozin, Eds. New York: Wiley, pp. 163–203, 1963.

[20] R. C. Jeffrey, *The Logic of Decision*. New York: McGraw-Hill, 1965.

[21] A. Kaufmann and M. Gupta, *Introduction to Fuzzy Arithmetic*. New York: Van Nostrand, 1985.

[22] E. P. Klement, W. Schwyhla, and R. Lowen, "Fuzzy probability measures," *Fuzzy Sets and Systems*, vol. 5, pp. 83–108.

[23] H. E. Kyburg, *Probability and Inductive Logic*. New York: McMillan, 1970.

[24] E. H. Mamdani and B. R. Gaines, *Fuzzy Reasoning and its Applications*. London: Academic, 1981.

[25] J. D. McCawley, *Everything that Linguists have Always Wanted to Know about Logic*. Chicago: University of Chicago, 1981.

[26] J. McCarthy, "Circumscription: A non-monotonic inference rule," *Artificial Intelligence*, vol. 13, pp. 27–40, 1980.

[27] D. V. McDermott and J. Doyle, "Non-monotonic logic—Part I," *Artificial Intelligence*, vol. 13, pp. 41–72, 1980.

[28] D. V. McDermott, "Non-monotonic logic—Part II: non-monotonic modal theories," *J. Assoc. Comp. Mach*. vol. 29, pp. 33–57, 1982.

[29] D. H. Mellor, *Matter of Chance*. Cambridge: Cambridge University, 1971.

[30] M. Mizumoto, S. Fukame, and K. Tanaka, "Fuzzy reasoning methods by Zadeh and Mamdani, and improved methods," *Proc. Third Workshop on Fuzzy Reasoning*, Queen Mary College, London, 1978.

[31] G. C. Moisil, *Lectures on the Logic of Fuzzy Reasoning*. Bucarest: Scientific Editions, 1975.

[32] R. C. Moore and J. R. Hobbs, Eds., *Formal Theories of the Commonsense World*. Harwood, NJ: Ablex, 1984.

[33] C. V. Negoita, *Expert Systems and Fuzzy Systems*. Menlo Park: Benjamin/Cummings, 1985.

[34] N. Nilsson, *Probabilistic Logic*, SRI tech. note 321, Menlo Park, CA, 1984.

[35] P. Peterson, "On the logic of few, many and most," *Notre Dame J. Formal Logic*, vol. 20, pp. 155–179, 1979.

[36] R. Reiter and G. Criscuolo, "Some representational issues in default reasoning," *Computers and Mathematics*, vol. 9, pp. 15–28, 1983.

[37] N. Rescher, *Plausible Reasoning*. Amsterdam: Van Gorcum, 1976.

[38] E. H. Shortliffe and B. Buchanan, "A model of inexact reasoning in medicine," *Mathematical Biosciences*, vol. 23, pp. 351–379, 1975.

[39] G. Soula and E. Sanchez, "Soft deduction rules in medical diagnostic processes," in *Approximate Reasoning in Decision Analysis*, M. Gupta, and E. Sanchez, Eds. Amsterdam: North-Holland, 1982, pp. 77–88.

[40] M. Sugeno, "Fuzzy measures and fuzzy integrals: A survey," in *Fuzzy Automata and Decision Processes*, M. Gupta, G. N. Saridis, and B. R. Gaines, Eds. Amsterdam: North-Holland, 1977, pp. 89–102.

[41] P. Szolovits and S. G. Pauker, "Categorical and probabilistic reasoning in medical diagnosis," *Artificial Intelligence*, vol. 11, pp. 115–144, 1978.

[42] R. R. Yager, "Quantified propositions in a linguistic logic," in *Proc. 2nd Int. Seminar Fuzzy Set Theory*, E. P. Klement, Ed. Linz, Austria: Johannes Kepler University, 1980.

[43] L. A. Zadeh, "Similarity relations and fuzzy orderings," *Info. Sci.*, vol. 3, pp. 177–200, 1976.

[44] L. A. Zadeh, "Fuzzy logic and approximate reasoning (in memory of Grigore Moisil)," *Synthese*, vol. 30, pp. 407–428, 1975.

[45] L. A. Zadeh, "A theory of approximate reasoning," tech. Memo. no. M77/58, University of California, Berkeley, 1977. See also *Machine Intelligence*, (vol. 9), J. E. Hayes, D. Michie, and L. I. Kulich, Eds. New York: John Wiley, 1979, pp. 149–194.

[46] L. A. Zadeh, "Test-score semantics for natural languages and meaning-representation via PRUF," tech. memo. no. 247, AI Center, SRI International, Menlo Park, CA, 1981. See also *Empirical Semantics*, B. B. Rieger, Ed. Bochum: Brockmeyer, 1981, pp. 281–349.

[47] L. A. Zadeh, "A computational approach to fuzzy quantifiers in natural languages," *Computers and Mathematics*, vol. 9, pp. 149–184, 1983.

[48] L. A. Zadeh, "The role of fuzzy logic in the management of uncertainty in expert systems," *Fuzzy Sets and Systems*, vol. 11, pp. 199–227, 1983.

[49] L. A. Zadeh, "A theory of commonsense knowledge," in *Issues of Vagueness*, H. J. Skala, S. Termini, and E. Trillas, Eds. Dordrecht: Reidel, 1984, pp. 257–296.

[50] L. A. Zadeh, "A computational theory of dispositions," in *Proc. 1984 Int. Conf. Computational Linguistics*, 1984, pp. 312–318.